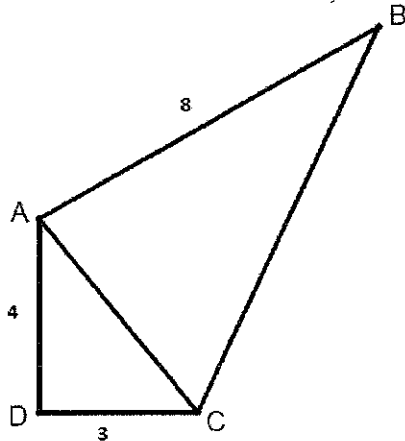


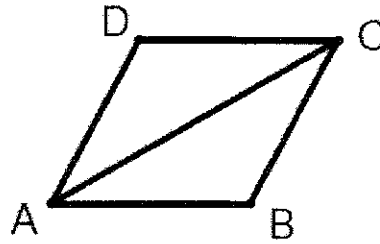
2016 John O'Bryan Mathematical Competition
Freshman-Sophomore Individual Test

Directions: Please answer all questions on the answer sheet provided. All answers must be written legibly in the correct blanks on the answer sheet and in simplest form. **Exact** answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

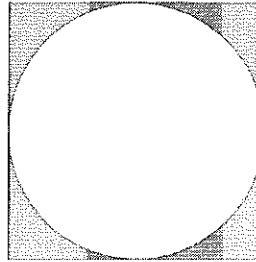
- (Select all that apply) For your answer write the **capital letter(s)** which corresponds to all correct choice. There are 9 sections of seats in an auditorium. Each section contains more than 89 seats and less than 157 seats. Which of the following could be the number of seats in this auditorium?
A) 98 B) 801 C) 809 D) 1056 E) 1405 F) 1695
- The slope of a line that contains points $A(6, k)$ and $B(11, w)$ is 2. Find the value of $(w - k)$.
- A certain number is six more than 80% of 80. Find the value of that number.
- Two cartons have weights of $(4x - 21)$ and $(7x + 11)$ respectively. Their average weight is 72 pounds. What is the **difference** in weights between the cartons?
- The two longer sides of a rectangle have a total length of 120, and one of the diagonals of this rectangle has a length of 65. Find the length of one of the shorter sides of the rectangle.
- Two cubes have edges whose respective lengths are 5 and 6. The ratio of the volume of the smaller cube to the volume of the larger cube is $k : w$, where k and w are positive integers. Find the smallest possible value of $(k + w)$.
- Given $A(2, 3)$, $B(4, 10)$, and $C(16, 7)$, find the **ordered pair** that represents the point at which the median from B intersects \overline{AC} .
- The graph of the function $y = x^2 - 2x + k$ has x -intercepts that are 6 units apart. Find the value of k .
- Suppose x and y are non-negative integers representing bases in the equation: $\left(\frac{24_x}{30_x}\right) = \left(\frac{3_y}{4_y}\right)$. Find the smallest possible base-ten value of $x + y$.
- In the diagram, $\overline{AD} \perp \overline{CD}$ and $\overline{AB} \perp \overline{AC}$. $AB = 8$, $AD = 4$, and $CD = 3$. Find the area of quadrilateral $ABCD$.



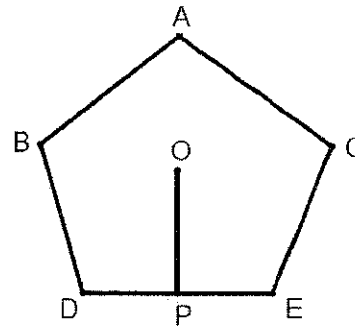
11. A trapezoid has sides with respective lengths 2, 15, 20, 15. Find the length of an altitude of this trapezoid.
12. In the diagram, $ABCD$ is a rhombus. If the degree measure of $\angle ADC$ is 7 times the degree measure of $\angle BAC$, find the degree measure of $\angle BCD$.



13. A circle is inscribed in a square of side length 1 unit. Find the area of the shaded region. Express your answer as a **decimal**, rounded to four places.



14. The sum of the digits of a three-digit positive integer is 18. The hundreds digit divides the remaining two-digit integer without remainder. The ones digit is 5 more than the tens digit. Find the three-digit integer.
15. Suppose that x , y , and z are positive integers with $8x + y + 12z = 62$. Find the minimum value of the **sum** $x + y + z$.
16. Let $ABCDE$ be a regular pentagon with center at point O . Point P lies on \overline{DE} such that $\overline{OP} \perp \overline{DE}$. If \overline{OE} were drawn, find the degree measure of $\angle EOP$.



17. Find the **sum** of the number of degrees in the interior angles of a convex polygon having 18 sides.
18. A bag has 4 red marbles and 2 blue marbles. You take out two marbles at the same time. Find the probability that they are the same color. Give your answer as a reduced common fraction.
19. Find the largest integer that divides 300, 417, and 764 with remainders R_1 , R_2 , and R_3 , respectively, such that $R_2 = R_1 + 3$ and $R_3 = R_2 + 5$.
20. In the city of Stormwind there are 405 druids and 459 warlocks. The fractional part of the total of these druids and warlocks that are warlocks can be expressed as $\frac{k}{w}$ where k and w are positive integers. Find the smallest possible value of $k + w$.

Name: _____

Team Code: _____

**2016 John O'Bryan Mathematical Competition
Freshman/Sophomore Individual Test**

Note: All answers must be written legibly in the correct blanks on the answer sheet and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

1. _____

11. _____

2. _____

12. _____

3. _____

13. _____

4. _____

14. _____

5. _____

15. _____

6. _____

16. _____

7. _____

17. _____

8. _____

18. _____

9. _____

19. _____

10. _____

20. _____

Name: _____ **ANSWERS** _____

Team Code: _____

**2016 John O'Bryan Mathematical Competition
Freshman-Sophomore Individual Test**

Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value (1 point).

1. **D**

11. **12**

2. **10**

12. **40** Degrees Optional

3. **70**

13. **0.2146** Must be this exact decimal

4. **74**

14. **927**

5. **25**

15. **8**

6. **341**

16. **36**

7. **(9,5)** Must be this ordered pair.

17. **2880** Degrees Optional

8. **-8**

18. **7/15** Must be this reduced fraction.

9. **21**

19. **114**

10. **26**

20. **49**

Awards Lists and Solutions to the Team Competition may be found at
<http://math.nku.edu/job>